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THE ALTITUDE OF STABILIZATION FOR SLOWLY TUMBLING RE-ENTRY VEHICLES

R. A. Norling

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~~ABSTRACT~~

An analysis is made to identify the parameters which determine the altitude of stabilization defined as the altitude at which the angle of attack rate first vanishes for slowly tumbling re-entry vehicles. The analysis is limited to small tumble rates and angles of attack which allow the assumptions of $C_m = C_{m_\alpha}\alpha$ and C_{m_α} = constant respectively. An expression is derived which implicitly relates the altitude of stabilization to (1) the initial descent velocity, $V_E \sin \gamma$, (2) the atmospheric scale height, (3) a vehicle stability parameter, $C_{m_\alpha} Al/I$, and (4) the quantity $(\alpha_o/\dot{\alpha}_o)$. The important result is the fact that the initial angle of attack and initial tumble rate, i.e., α_o and $\dot{\alpha}_o$ respectively, appear only as a ratio, $\alpha_o/\dot{\alpha}_o$, the remaining parametric quantities being properties of the vehicle, the atmosphere, and the trajectory.

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NOMENCLATURE

A	=	characteristic area, ft^2
c	=	inverse scale height = $1/17,150\ ft^{-1}$, assumed constant
C_1, C_2	=	constants of integration; see Eqs. (9a) and (9b)
C_m	=	aerodynamic moment coefficient
C_{m_α}	=	$(dC_m/d\alpha)$, assumed constant
h	=	altitude, ft.
I	=	pitch moment of inertia, slugs-ft^2
$J_o(x)$	=	Bessel function of first kind of order zero
$J_o'(x)$	=	$(dJ_o(x)/dx)$
l	=	reference aerodynamic length. ft.
n	=	tumble rate, rpm
P_s	=	stability parameter = $C_{m_\alpha} Al/I$ (ft/slug)
q	=	dynamic pressure
V	=	velocity, assumed constant
x	=	defined by Eq. (4)
$Y_o(x)$	=	Bessel function of second kind of order zero
$Y_o'(x)$	=	$(dY_o(x)/dx)$
α	=	angle of attack
$\dot{\alpha}$	=	$d\alpha/dt$
α'	=	$d\alpha/dx$
γ	=	re-entry path angle, assumed constant
λ	=	Euler's constant = 1. 78
ρ	=	atmospheric density = $\rho_R e^{-ch}$
ρ_R	=	reference density = . 156 slug/ft^3

ω_a = aerodynamic oscillation frequency = $\sqrt{qP_s}$ rad/sec.

Subscripts

0 = initial re-entry conditions

1 = conditions at $\dot{\alpha} = 0$ first time

2 = conditions at $\dot{\alpha} = 0$ second time

ANALYSIS

When a re-entry vehicle is traversing the altitude regime during which the aerodynamic forces are beginning to become strong enough to influence body motion, the analysis of such motions can be made with considerable simplification. These simplifications are itemized below and will be incorporated in the analysis to follow:

(1) The path angle is relatively constant.

(2) The velocity is nearly constant.

(3) Aerodynamic damping can be neglected.

Furthermore, this analysis assumes that the re-entry vehicle is: (a) non-spinning, (b) tumbling in a plane which contains the flight velocity vector, (c) not experiencing normal forces, (d) oscillating with a linear moment coefficient; i. e., C_{m_α} is a constant. Under the preceding assumptions, the differential equation of motion becomes:

$$\ddot{\alpha} = \frac{\rho V^2}{2}\left(\frac{C_{m_\alpha} A l}{I}\right)\alpha \qquad (1)$$

Equation (1) can be rewritten as:

$$\ddot{\alpha} + \left(\frac{\rho V^2}{2} P_s\right)\alpha = 0 \qquad (1a)$$

where P_s = stability parameter = $-(C_{m_\alpha} Al/I)$.

The stability parameter reflects both geometric and mass characteristics of the re-entry vehicle. Since only stable vehicles with linear restoring moments will be treated, the stability parameter will always be a positive number. The frequency of oscillation or "aerodynamic frequency" is given immediately by:

$$\omega_a^2 = \left(\frac{\rho V^2}{2}\right) P_s = q P_s$$

Therefore, the instantaneous aerodynamic frequency squared is a product of the dynamic pressure and stability parameter and is similar to the expression previously derived in Ref. 1. Following the usual custom, we assume the density can be written as:

$$\rho = \rho_R e^{-ch} \tag{2}$$

where ρ_R = reference value of density

c = inverse scale height

The calculations contained in this report were made utilizing values of $c = 1/17,150\ ft^{-1}$ and $\rho_R = .156\ slugs/ft^3$, which are reasonable approximations over the altitude ranges of interest as shown in Fig. 1.

At this point, it is convenient to make a change of independent variable. The one dimensional equation in terms of time is rewritten in the form:

$$\ddot{\alpha} + \omega_a^2(t)\alpha = 0 \tag{3}$$

Define a new variable, $x = 2\,\omega_a/cV \sin\gamma$, (4)

where is the re-entry path angle shown in Fig. 2. We then proceed to transform the independent variable.

$$\frac{d\alpha}{dt} = \frac{d\alpha}{dx}\frac{dx}{d\rho}\frac{d\rho}{dt}$$

$$\frac{dx}{d\rho} = \frac{dx}{d\omega_a}\frac{d\omega_a}{d\rho} = \frac{\omega_a}{c\rho V \sin\gamma}$$

$$\frac{d\rho}{dt} = \frac{d\rho}{dh}\frac{dh}{dt} = (-c\rho)(-V\sin\gamma) = c\rho V \sin\gamma$$

Hence, upon substituting we obtain:

$$\frac{d\alpha}{dt} = \omega_a \frac{d\alpha}{dx}$$

$$\frac{d^2\alpha}{dt^2} = \omega_a \frac{d^2\alpha}{dx^2}\frac{dx}{dt} + \frac{d\alpha}{dx}\frac{d\omega_a}{dx}\frac{dx}{dt}$$

$$\frac{d\omega_a}{dx}\frac{dx}{dt} = \left(\frac{cV\sin\gamma}{2}\right)\omega_a = \frac{\omega_a^2}{x}$$

Therefore, the desired result becomes:

$$\frac{d^2\alpha}{dt^2} = \omega_a^2 \frac{d^2\alpha}{dx^2} + \frac{\omega_a^2}{x}\frac{d\alpha}{dx} \tag{5}$$

Substitution of Eq. (5) into (3) yields the transformed equation of motion.

$$\alpha''(x) + \frac{1}{x}\alpha'(x) + \alpha(x) = 0 \tag{6}$$

The solution of Eq. (6) immediately can be written as

$$\alpha(x) = C_1 J_0(x) + C_2 Y_0(x) \tag{7a}$$

$$\alpha'(x) = C_1 J_0'(x) + C_2 Y_0'(x) \tag{7b}$$

Jo(x) and Yo(x) are the Bessel functions of the first and second kinds respectively of order zero.

The initial conditions on the motion are prescribed as:

$$\alpha(0) = \alpha_0$$

$$\alpha'(0) = \alpha_0'$$

where $d\alpha/dt$ and $d\alpha/dx$ are related by the expression

$$\alpha' = \frac{d\alpha}{dx} = \frac{1}{\omega_a}\frac{d\alpha}{dt} = \frac{\dot{\alpha}}{\omega_a}$$

The initial re-entry conditions become:

$$\alpha(0) = \alpha_0 \tag{8a}$$

$$\alpha'(0) = \frac{\dot{\alpha}_0}{\omega_{a_0}} \tag{8b}$$

The constants C_1 and C_2 in Eqs. (7a) and (7b) can be evaluated from the initial conditions given in (8a) and (8b) plus the initial value of the quantity defined in (4), i.e., $x_0 = 2\,\omega_{a_0} / cV \sin\gamma$.

Solving the determinant yields the following values for C_1 and C_2:

$$C_1 = \frac{\alpha_0 Y_0'(x_0) - \frac{\dot{\alpha}_0}{\omega_{a_0}} Y_0(x_0)}{J_0(x_0) Y_0'(x_0) - J_0'(x_0) Y_0(x_0)} \tag{9a}$$

$$C_2 = \frac{\frac{\dot{\alpha}_0}{\omega_{a_0}} J_0(x_0) - \alpha_0 J_0'(x_0)}{J_0(x_0) Y_0'(x_0) - J_0'(x_0) Y_0(x_0)} \tag{9b}$$

Equations (9a) and (9b) in conjunction with equations (7a) and (7b) constitute the solution.

The initial tumble rate, i.e., $\dot{\alpha}_0$, is restricted to values which permit the linear theory to remain valid. Very high tumble rates would cause the re-entry vehicle to flip over or at best cause the angle of attack to become sufficiently large to invalidate the assumption of C_{m_α} = constant, since for most practical re-entry shapes the linearity of the restoring moment is preserved only for small or moderate angles of attack. The precise limitations on the magnitude of $\dot{\alpha}_0$ (corresponding to some initial altitude h_0) cannot be determined a priori. One must first make the calculation to determine the angle of attack excursion (e.g., Fig. 5) for a particular set of initial re-entry conditions to insure that the linear approximations are satisfied.

The initial altitude (h_0) is reflected in the quantity ω_{a_0}, which in turn appears in the final expressions as x_0. The quantities α_0 and $\dot{\alpha}_0$ define the initial conditions regarding body attitude, while the quantity x_0 reflects the initial altitude and descent velocity. In practice, one must be careful to choose an initial altitude sufficiently high so that the aerodynamic restoring moments (for the given stability parameter) are initially negligible. At the same time, the initial altitude must not be so high that the integral $\int_0^t \dot{\alpha}_0\,dt$ becomes excessively large before the aerodynamic restoring moments dominate the motion, i.e., before the body flips or the angle becomes too large to permit the assumption C_{m_α} = constant.

The initial tumble rate, i.e., $\dot{\alpha}_o$, is restricted to values which permit the linear theory to remain valid. The limiting parameter for the planar case generally is the term C_{m_α}, which was assumed constant. In reality, the term will vary over large ranges of α. The degree of non-linearity is dependent upon the particular re-entry vehicle shape. The calculations to be discussed later utilized the equations of (7a) and (7b).

APPROXIMATE SOLUTION

Equations (7a), (7b), (9a), and (9b) are extremely untractable in their current form. In most instances, the initial conditions occur at sufficiently high altitudes to permit simplifications which arise from the fact that $x_o \ll 1$.

For $x_o \ll 1$, we may state the following:[2]

$$J_o(x_o) \cong 1$$

$$J_o'(x_o) \cong -\frac{x_o}{2} J_o(x_o) \cong -\frac{x_o}{2}$$

$$Y_o(x_o) \cong \frac{2}{\pi} J_o(x_o) \ln \frac{\lambda x_o}{2} \cong \frac{2}{\pi} \ln \frac{\lambda x_o}{2}$$

$$Y_o'(x_o) \cong J_o(x_o)\left[\frac{2}{\pi x_o} - \frac{x_o}{\pi} \ln \frac{\lambda x_o}{2}\right] \cong \frac{2}{\pi x_o}\left[1 + \frac{x_o^2}{2} \ln \frac{2}{\lambda x_o}\right]$$

where λ = Euler's constant = 1.78.

Using the above approximations, the constants C_1 and C_2 may be rewritten as:

$$C_1 = \left[\alpha_o\left(1 + \frac{x_o^2}{2} \ln \frac{1.124}{x_o}\right) + \frac{2\dot{\alpha}_o}{cV\sin\gamma} \ln \frac{1.124}{x_o}\right] \cong \alpha_o + \frac{2\dot{\alpha}_o}{cV\sin\gamma} \ln \frac{1.124}{x_o}$$

$$C_2 = \pi\left[\frac{\dot{\alpha}_o}{cV\sin\gamma} + \frac{\alpha_o x_o^2}{4}\right] \cong \frac{\pi\dot{\alpha}_o}{cV\sin\gamma}$$

The expressions for angle of attack and angle of attack rate become:

$$\alpha = \left[\alpha_o + \frac{2\dot{\alpha}_o}{cV\sin\gamma} \ln\frac{1.124}{x_o}\right] J_o(x) + \frac{\pi\dot{\alpha}_o}{cV\sin\gamma} Y_o(x) \tag{10a}$$

$$\dot{\alpha} = \omega_a \left\{ \left[\alpha_o + \frac{2\dot{\alpha}_o}{cV\sin\gamma} \ln\frac{1.124}{x_o}\right] J_o'(x) + \frac{\pi\dot{\alpha}_o}{cV\sin\gamma} Y_o'(x) \right\} \tag{10b}$$

It is desired to determine the altitude at which the re-entry vehicle stabilizes, i.e., the altitude at which $\dot{\alpha} = 0$ for the first time. Designate h_1 as the altitude of the first peak of α ($\dot{\alpha} = 0$ first time) and h_2 as the corresponding position of the second peak of α ($\dot{\alpha} = 0$ second time). These can be determined by setting $\dot{\alpha} = 0$ in Eq. (10b).

$$\left[\alpha_o + \frac{2\dot{\alpha}_o}{cV\sin\gamma} \ln\frac{1.124}{x_o}\right] J_o'(x) + \frac{\pi\dot{\alpha}_o}{cV\sin\gamma} Y_o'(x) = 0$$

or

$$\frac{Y_o'(x)}{J_o'(x)} + \frac{cV\sin\gamma}{\pi}\left(\frac{\alpha_o}{\dot{\alpha}_o}\right) + \frac{2}{\pi}\ln\frac{1.124}{x_o} = 0 \tag{11}$$

This equation can be solved for the quantity x in terms of the initial conditions. The significance of Eq. (11) is that the initial angle of attack and tumble rate appear as a parameter $(\alpha_o/\dot{\alpha}_o)$. The $\dot{\alpha}_o$ quantity is assumed finite in Eq. (11). A similar expression could be derived in terms of $(\dot{\alpha}_o/\alpha_o)$. We can conclude from Eq. (11) that the altitude of stabilization for slowly tumbling re-entry vehicles may be expressed as a function of $(\alpha_o/\dot{\alpha}_o)$, in addition to the initial value x_o, which reflects initial altitude and initial descent velocity.

RESULTS

The altitudes at which the angle of attack rate vanishes, i.e., $\dot{\alpha} = 0$, were calculated from Eq. (7b) and plotted in Figs. 3 and 4 as a function of α_o/n in the manner suggested by Eq. (11). The quantity n is taken as the tumble rate in rpm. Also plotted in Figs. 3 and 4 are the results of a direct integration of Eq. (1a) by the IBM 650 machine using the ARDC atmosphere, as indicated by the circles. Fig. 3 indicates the altitudes (h_1) at which $\dot{\alpha} = 0$ for the first time as a parametric function of

P_s, assuming an initial descent velocity of 8000 fps and an initial altitude of 400 kilofeet. The roots of Equation (7b), i. e., $C_1 J_o'(x) + C_2 Y_o'(x) = 0$, were determined by an IBM 650 digital computer. Although the approximate solution given by Eq. (11) also would provide an accurate determination of the argument x at which $\dot{\alpha} = 0$, the chief purpose of that result was to suggest the manner in which results could be correlated. It is noted that the stabilization altitudes are lowered by 20 to 25 kilofeet as the stability parameter is dropped by a factor of 5. Fig. 4 presents the altitudes of the second peak, i. e., $\dot{\alpha} = 0$ second time in a similar manner. The results of Fig. 3 show a wide variation in h_1 as α_o/n ranges between ± 200 deg/rpm, while Fig. 4 indicates that the altitude of the second peak, h_2, is less sensitive to the value of α_o/n. For values of $(\alpha_o/n) \ll 0$, the h_1 altitude approaches the no-tumble case asymptotically. Similarly, the h_2 altitudes approach the h_1 value for $(\alpha_o/n) \ll 0$. Therefore, slow tumble will always <u>increase</u> the altitude at which the body stabilizes.

Figure 5 presents the peak angle of attack (at h_1) vs. the initial angle of attack for positive and negative tumble rates and $P_s = 1/2$. The calculations were carried out for angles of attack, some of which in practical situations would occur in non-linear regions of the moment curve. Fig. 5 is applicable to linear moment curves only, i. e., $C_m = C_{m_\alpha} \alpha$. Fig. 6 indicates the variation of maximum angle of attack (α_1) as a function of P_s for differing tumble rates where an initial angle of attack of 30^o was chosen.

In Fig. 7 is shown the influence of the descent velocity, i. e., V sin γ on the altitude of the first peak for various values of the α_o/n parameter. For $\alpha_o/n = 0$, the altitude of first peak is seen to be independent of descent velocity.

CONCLUSIONS

The linearized analysis of body dynamics in early phases of reentry for small tumble rates leads to the conclusion that the altitude at which the angle of attack rate vanishes can be correlated with the quantity $(\alpha_o/\dot{\alpha}_o)$, having first defined an initial altitude, a stability parameter, and the initial descent velocity. Further, any finite tumble rate acts to raise the altitude of stabilization defined, as the altitude at which $\dot{\alpha}$ vanishes for the first time. The altitude of stabilization is reduced with decreasing values of P_s with the incremental decrease amounting to approximately 25 kilofeet per factor of 5 decrease in stability parameter over the range of P_s investigated; i. e., $1/50 \leq P_s = C_{m_\alpha} Al/I \leq 1/2$, assuming an initial descent velocity of 8000 fps. The variation of initial descent velocity tends to raise or lower the altitude of first peak depending on the sign of the quantity α_o/n. For $\alpha_o/n = 0$, there is no variation of h_1 with V sin γ. For descent velocities between 6000 and 10,000 fps, the altitudes of first peak vary from 6 to 10 kilofeet for $\alpha_o/n = \pm 50$ deg/rpm.

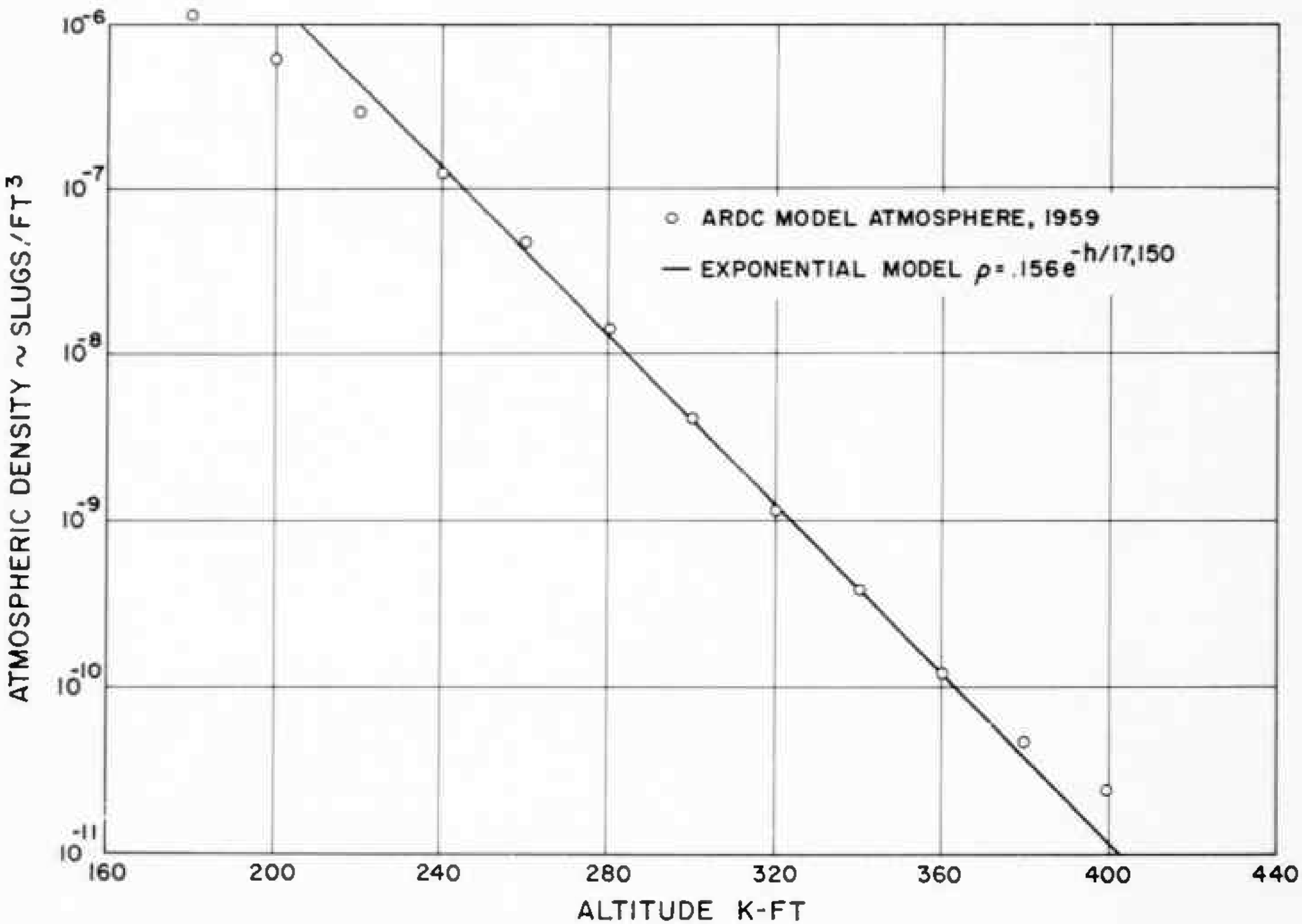

Fig. 1 Comparison of ARDC 1959 model atmosphere with exponential model used for calculations in this paper.

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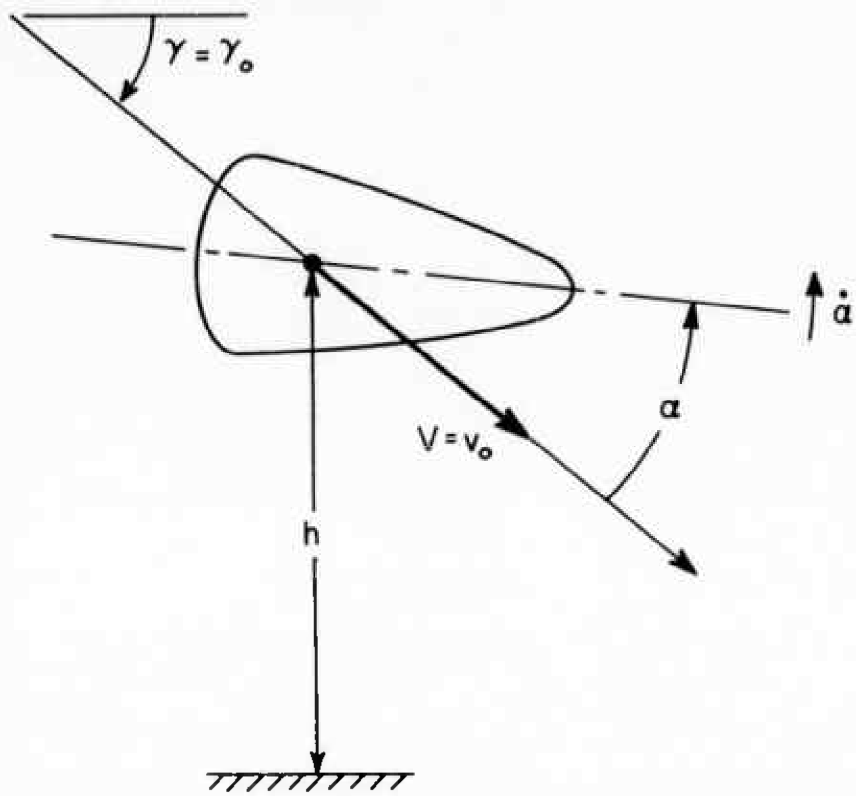

Fig. 2 Re-entry geometry.

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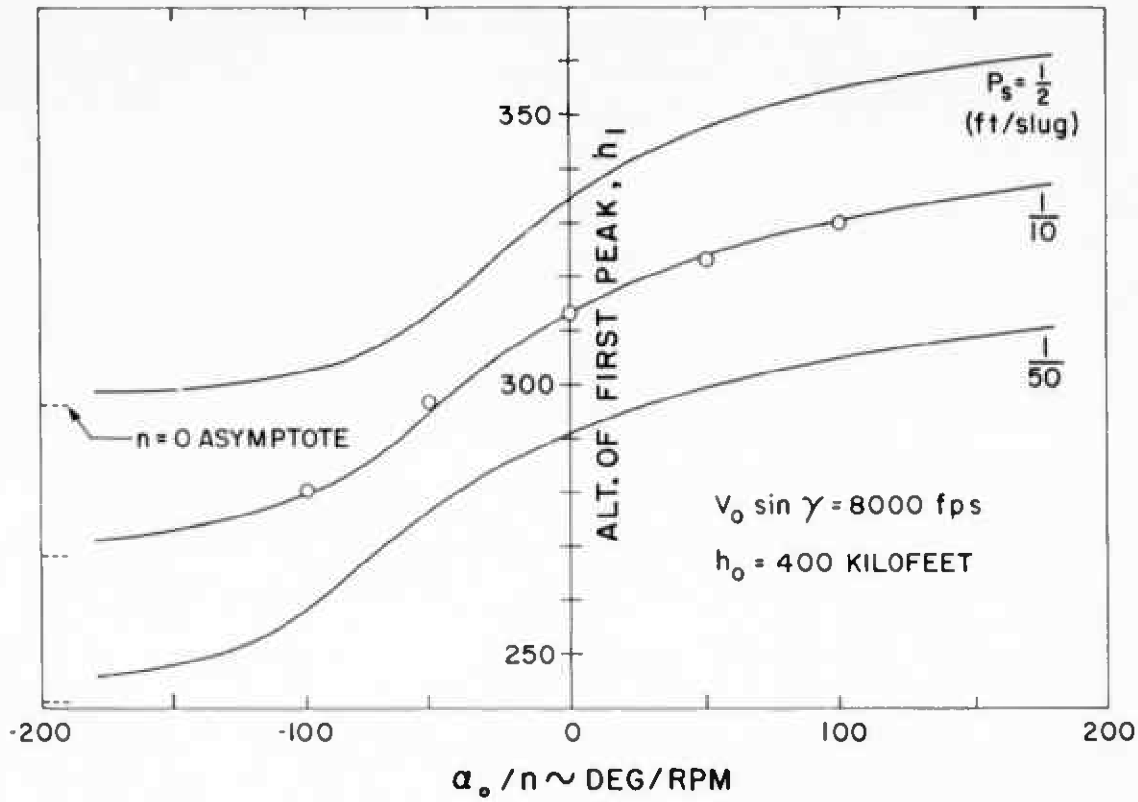

Fig. 3 Stabilization altitudes. This graph presents the altitudes of stabilization vs. (α_0/n) for various values of the stability parameter P_s. Circles indicate results of exact integration of Eq. (1a).

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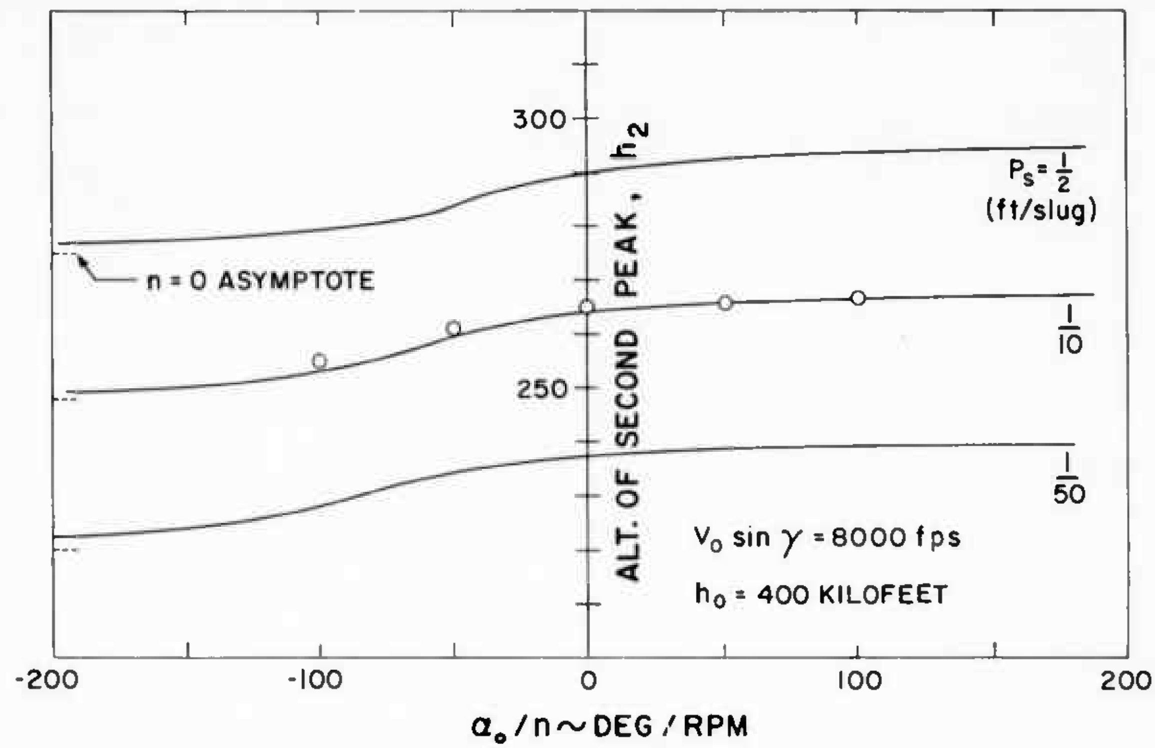

Fig. 4 Altitudes of second peak. This graph presents the altitudes at which the angle of attack rate vanishes the second time. Circles indicate results of exact integration of Eq. (1a).

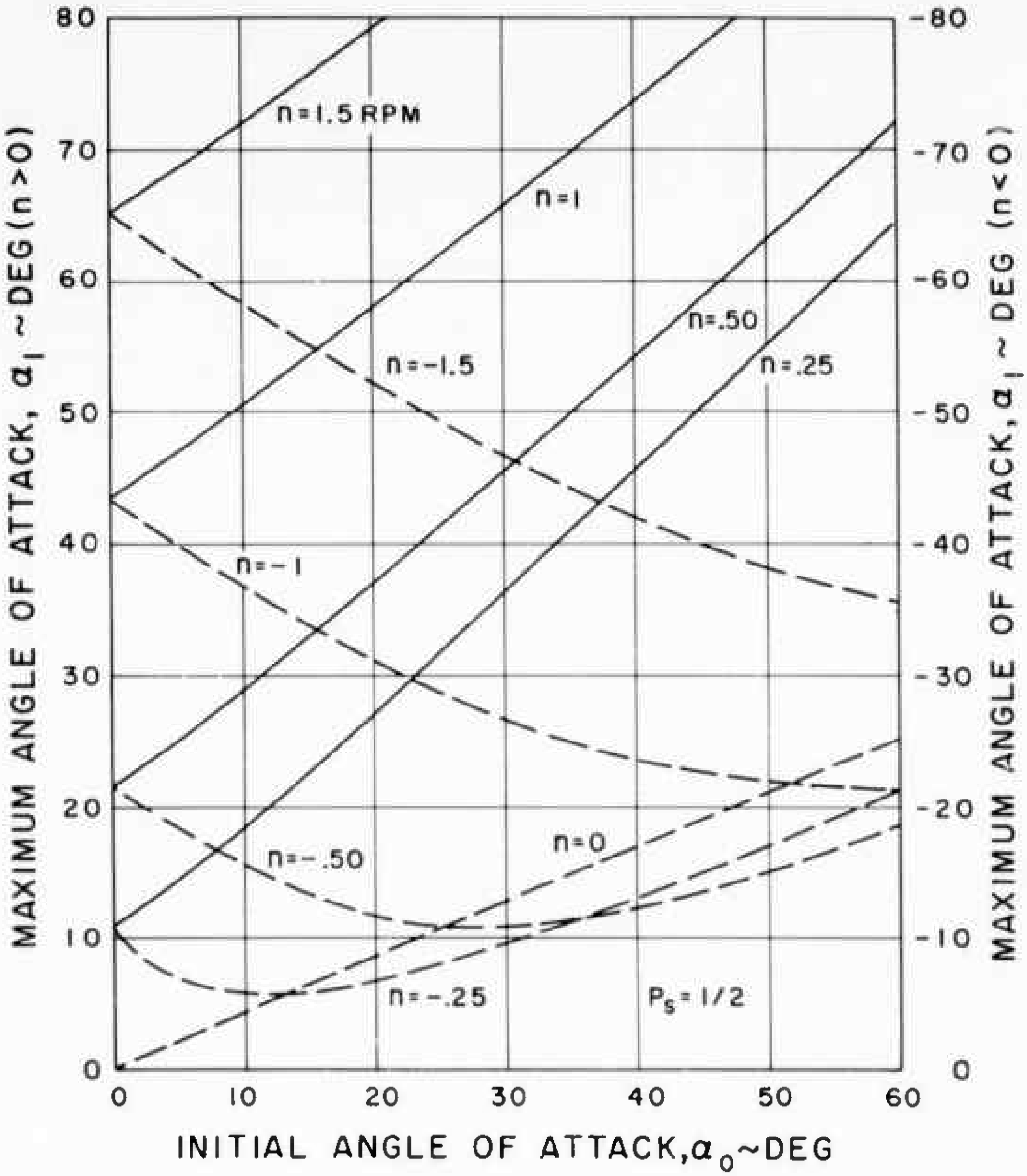

Fig. 5 Maximum angles of attack. This graph indicates the magnitude of the angle of attack at stabilization altitude as a function of initial angle of attack for $P_s = 1/2$.

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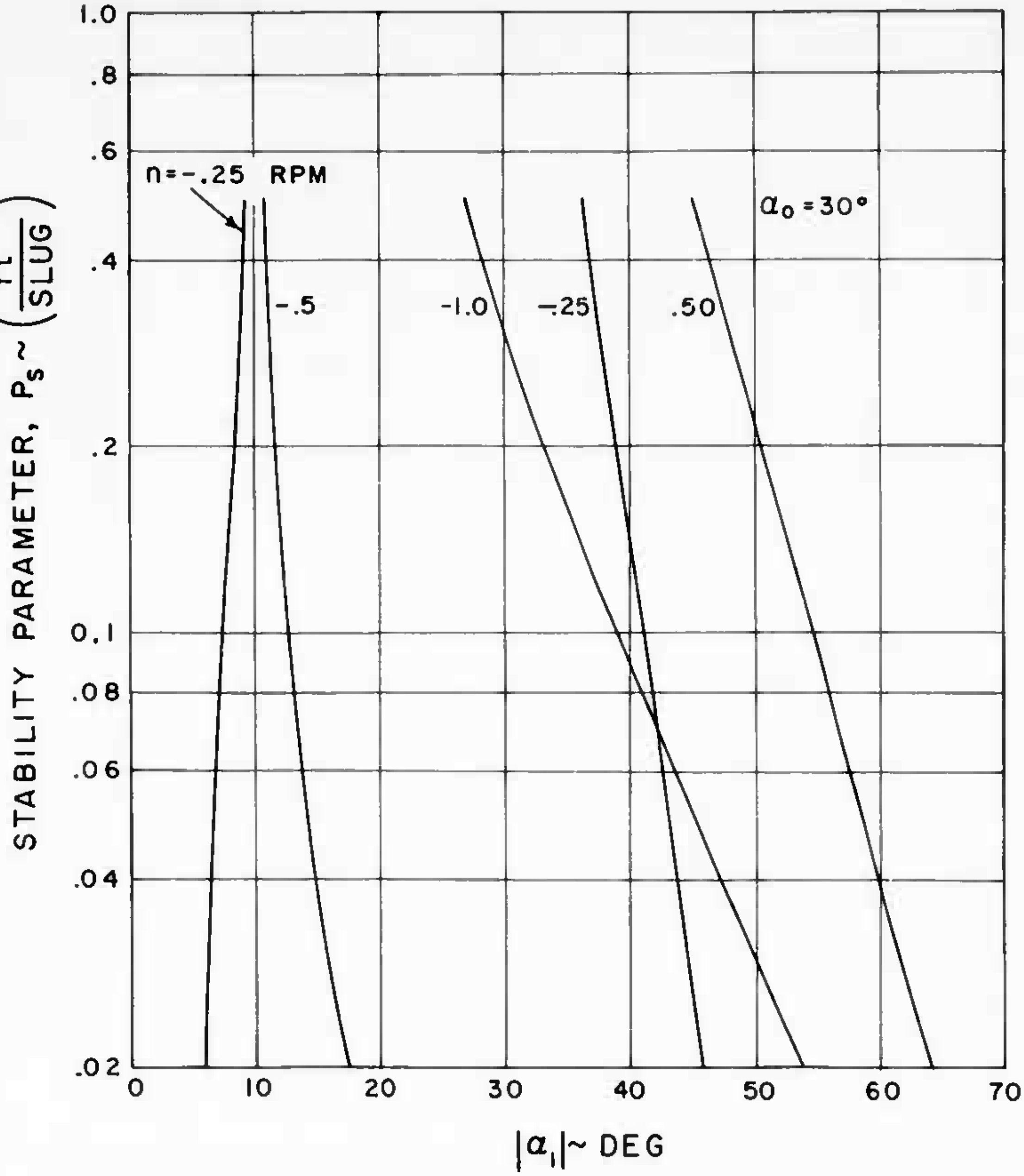

Fig. 6 Effect of stability parameter on $|\alpha_1|$. These curves illustrate the effect of P_s on $|\alpha_1|$ for an initial angle of attack of 30°.

5710

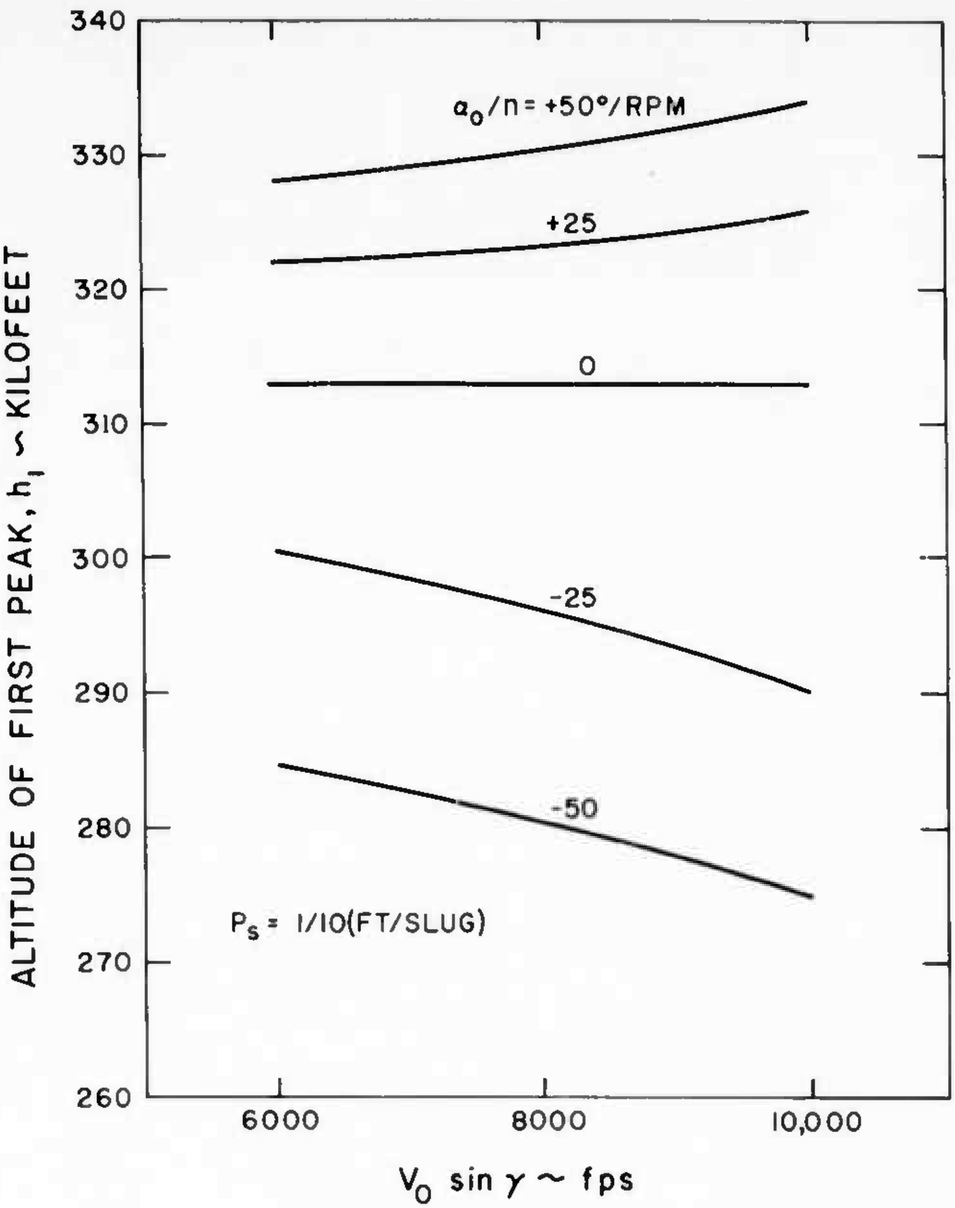

Fig. 7 Effect of descent velocity on altitude of first peak. This graph illustrates the effect of descent velocity on h_1 for various values of a_0/n, assuming $P_s = 1/10$.

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ratio, $a_o/\dot{a}_o$, the remaining parametric quantities being properties of the vehicle, the atmosphere, and the trajectory.

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1. Re-entry vehicles-Stability.
I. Title.
II. Norling, R. A.
III. Avco-Everett Research Report 134.
IV. AFBSD-TN-61-27.
V. Contract AF 04(647)-278.

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Avco-Everett Research Laboratory, Everett, Massachusetts
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